

Interaction of radiation with matter

- Alpha radiation, electrons and positrons, gamma radiation, etc. What we are going to study now is the ability of these particles to ionise the materials they pass through.
- Heavy, charged particles lose energy through collisions with electrons (Coulomb scattering) rather than with nuclei (Rutherford scattering), owing to the extremely small volume of the nuclei (10^{-15} of the total volume).
- After a large number of collisions the particle stops. The distance over which this happens is called **range**.

Tracks of alpha particles in a “cloud chamber” (*cloud chamber*) – note the well-defined range

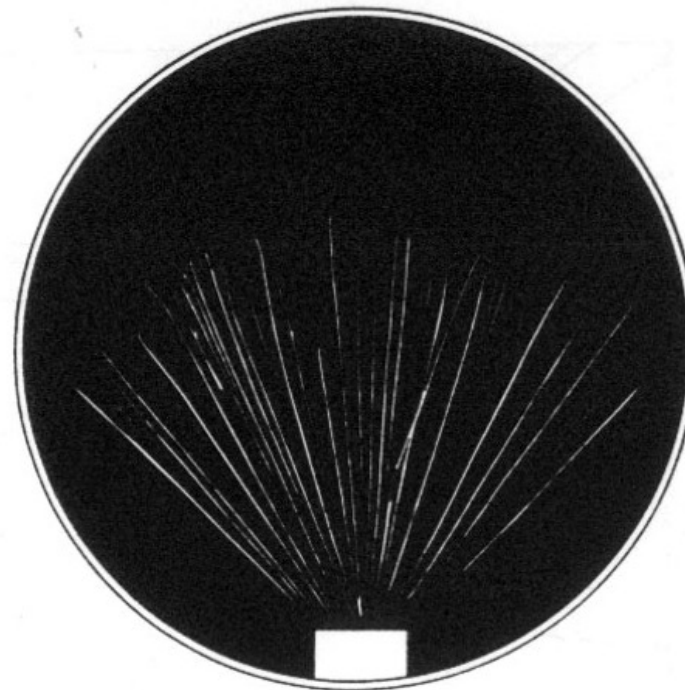
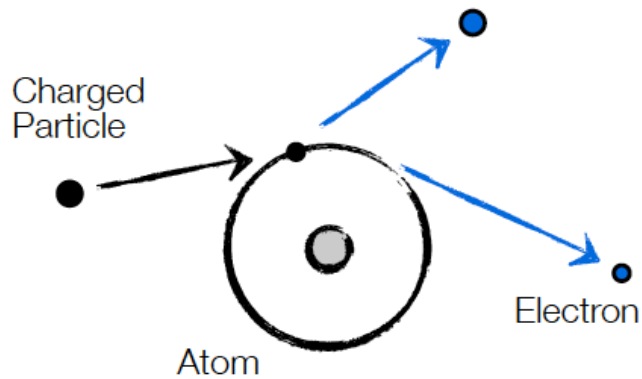
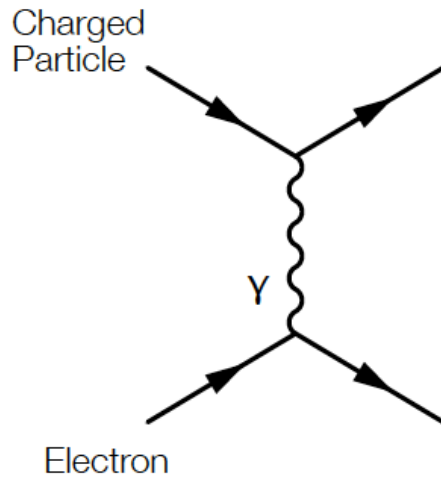


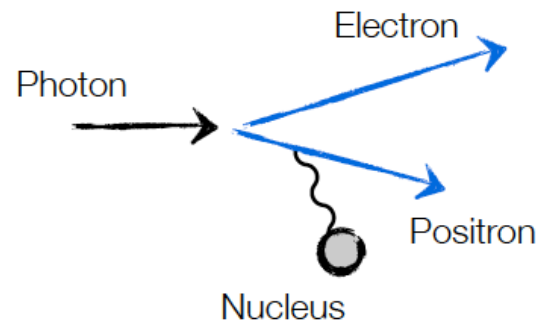
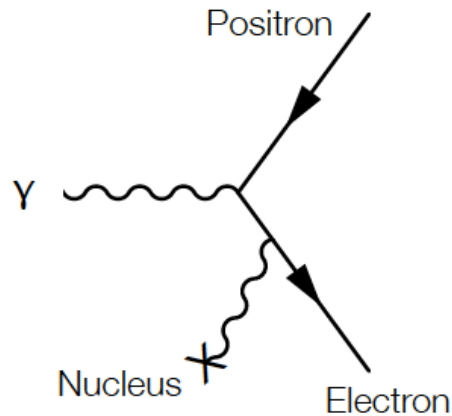
Figure 7.1 Cloud chamber tracks of α particles from the decay of ^{210}Po .

Some examples of interaction between particles

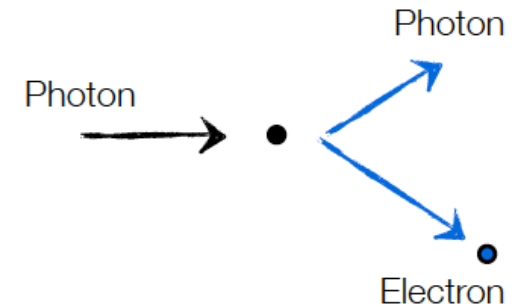
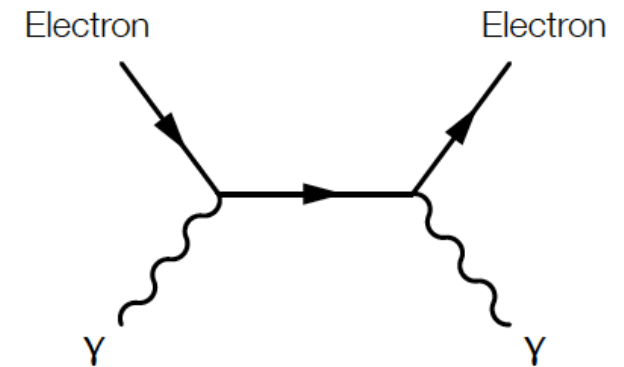
Ionization:



Pair production:



Compton scattering:



1. Alpha radiation – heavy, charged particles

Alpha particles are emitted with approximately 5 MeV. The energy losses per collision with an electron are of the order of 3 keV.

- a) Many thousands of collisions are needed for a particle to lose all its energy**
- b) Since the alpha particle is much heavier than the electron, it undergoes small deflections, following an approximately straight line.**
- c) Since the Coulomb force has infinite range, the particles lose energy continuously along their path. One usually works with a **mean range**, defined as the path length for which half the particles in the beam have greater ranges and the other half have shorter ranges.**
- d) The energy needed to ionise an atom (remove an electron) is of the order of 10 eV – so many atoms will be ionised. Some of the released electrons may produce further ions, thereby releasing more electrons (called secondaries). To find the energy lost by the particle we must take all these electrons into account.**

Bethe formula – (“stopping power”) - energy loss of heavy, charged particles, per unit length

$$-\frac{dE}{dx} = \frac{4\pi}{m_e c^2} \cdot \frac{n z^2}{\beta^2} \cdot \left(\frac{e^2}{4\pi\epsilon_0} \right)^2 \cdot \left[\ln \left(\frac{2m_e c^2 \beta^2}{I \cdot (1 - \beta^2)} \right) - \beta^2 \right]$$

$\beta = v/c$;

I – mean excitation/ionisation energy
of the atoms of the material; $I = 34$ eV for
air, and $I = 163$ eV for aluminium.

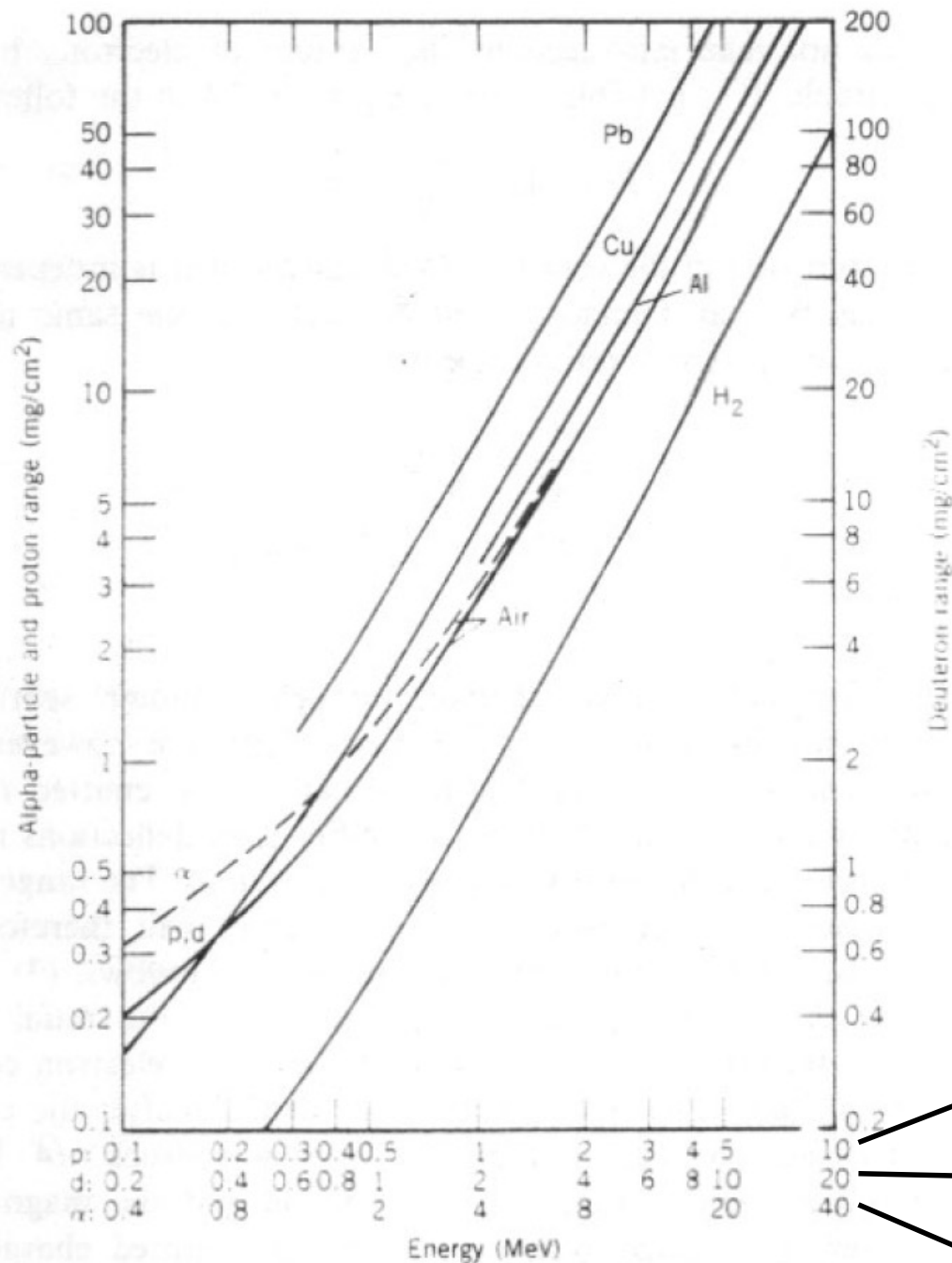
m_e - electron mass;

z – electric charge of the ionising particle, in units of the electron charge (dimensionless quantity);

ρ – density of the material;

n – number of electrons per unit volume of the material;

$n = N_A Z \rho / (M)$, where N_A is Avogadro's number; Z is the atomic number of the material; its mass density, and M its molar mass.



Range–energy relation for various materials. Since particles lose energy through scattering off atomic electrons, the range varies inversely with density. It therefore makes more sense to present the product range × density (this product is also called the range in the literature).

When alpha particles stop, they capture **electrons and turn into **Helium**.**

Proton

Deuteron

Alpha

Example of applying the Bethe formula for the case of air and lead, for an alpha particle with an energy of 4 MeV:

$$\rho_{\text{air}} = 1.22 \text{ kg/m}^3 = 1.2 \text{ mg/cm}^3$$

$$\rho_{\text{lead}} = 11.34 \text{ g/cm}^3 = 1.134 \times 10^4 \text{ mg/cm}^3$$

From the graph we read off

$$(\rho R)_{\text{chumbo}} = 8 \text{ mg/cm}^2 \leftarrow$$

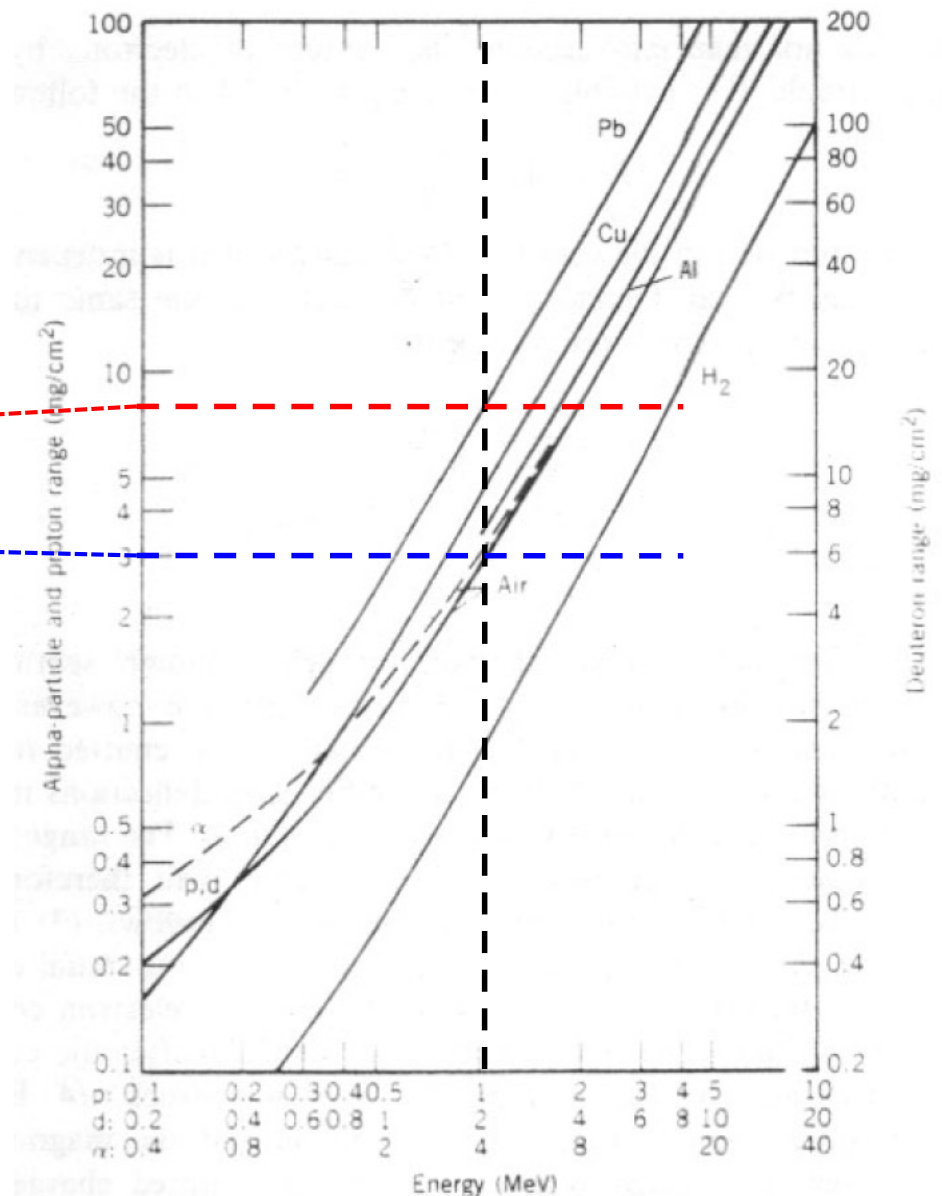
$$(\rho R)_{\text{ar}} = 3 \text{ mg/cm}^2 \leftarrow$$

And so the range is

$$R_{\text{chumbo}} = 8 / (1.134 \times 10^4) = 7.1 \times 10^{-4} \text{ cm}$$

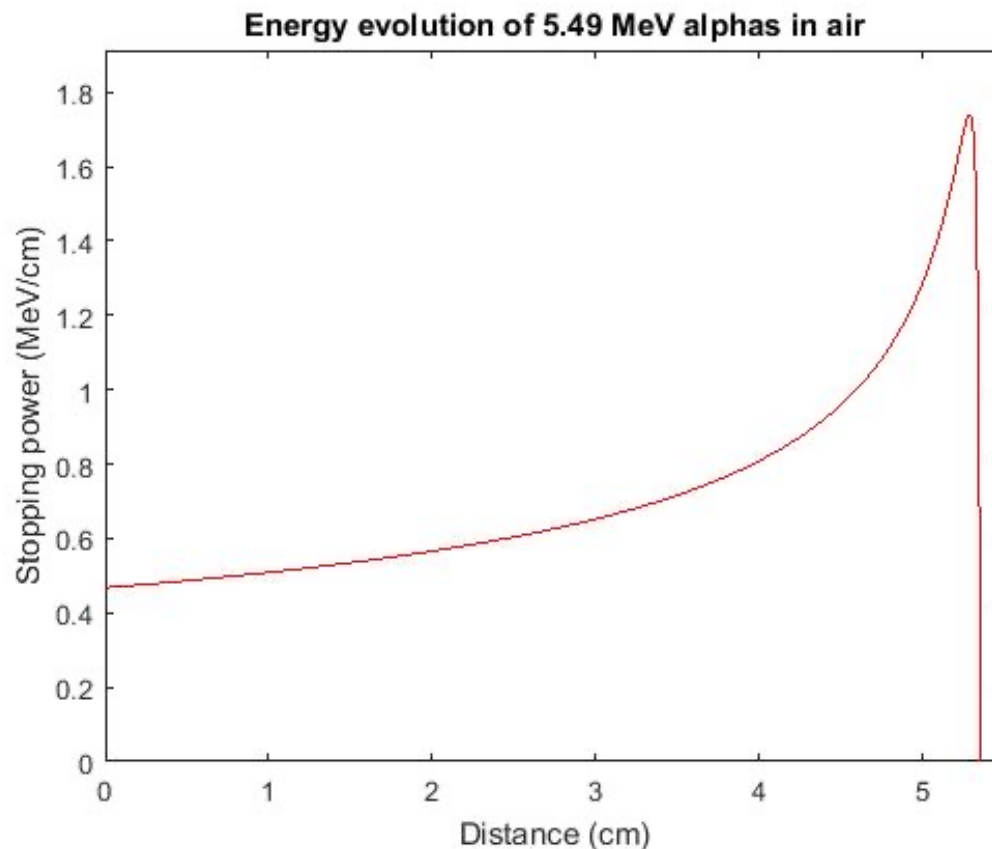
$$R_{\text{ar}} = 3 / (1.2) = 2.5 \text{ cm}$$

The difference between the values of R and ρR for the two cases nicely illustrates the usefulness of the variable ρR .



BRAGG PEAK

Applying the Bloch formula, as for alphas or protons, we see that **the way these particles lose energy is not uniform: the particles lose an appreciably constant amount of energy over a certain distance, before releasing a large amount of energy over a very short distance just before coming to rest.**



This maximum of deposited energy (note that it occurs over a very short spatial interval) is given the name **of *Bragg peak***, and it has notable medical applications (see ***proton therapy***).

Bragg-Kleeman rule

In a material with density ρ_0 and mass number A_0 the range is R_0 . In another material with density ρ_1 and mass number A_1 the range is R_1 . The ratio between the ranges in the two materials is given approximately by

$$\frac{R_1}{R_0} = \frac{\rho_0}{\rho_1} \sqrt{\frac{A_1}{A_0}}$$

In the same material, different radiations (with different masses and charges) have ranges given by

$$\frac{R_1}{R_0} = \frac{M_1}{M_0} \frac{z_0^2}{z_1^2}$$

where M is the particle's mass and z the charge in units of the electron charge.

$$\frac{R_{\alpha}}{R_{\text{protão}}} = \frac{4}{1} \frac{1}{4} = 1$$

This holds for particles with the same initial velocity. Since they are non-relativistic, the kinetic energies in this case differ by a factor of 4.

2. Beta radiation – light, charged particles (electrons and positrons)

Beta radiation can lose energy:

a) through **collisions**

b) through **radiation** (emission of *bremsstrahlung*, “braking radiation”)

Like alpha particles, electrons (and positrons) also interact with atomic electrons, but with some differences:

a) electrons travel at speeds close to that of light;

b) the deflections undergone are very large and the electrons' path through the material is erratic, i.e. the path length travelled and the range are very different;

c) the amount of energy transferred can be a very large fraction of the initial energy;

d) since the electron can undergo abrupt changes in direction and speed, it experiences large accelerations, and accelerated charged particles emit electromagnetic radiation called *bremsstrahlung* (from the German: braking radiation).

NOTE: positrons have another way of losing energy – when they stop, they annihilate with electrons in the medium and produce two 511 keV photons (**pair annihilation**)

Range of electrons

$$-\left(\frac{dE}{dx}\right)_{col} = \left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \frac{2\pi n}{m_e c^2 \beta^2} \left[\log \frac{T(T + m_e c^2)^2 \beta^2}{2I^2 m_e c^2} + 1 - \beta^2 - \left(2\sqrt{1 - \beta^2} - 1 + \beta^2\right) \log 2 + \frac{1}{8} \left(1 - \sqrt{1 - \beta^2}\right)^2 \right]$$
$$-\left(\frac{dE}{dx}\right)_{rad} = \left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \frac{nZ(T + m_e c^2)}{137m_e^2 c^4} \left[4 \log \frac{2(T + m_e c^2)^2 \beta^2}{m_e c^2} - \frac{4}{3} \right]$$

The subscripts *col* and *rad* refer to energy losses through collisions and through radiation respectively.

$\beta = v/c$;

I – excitation/ionisation energy of the atoms of the material;

m_e - electron mass;

ρ – density of the material; T – kinetic energy of the electron;

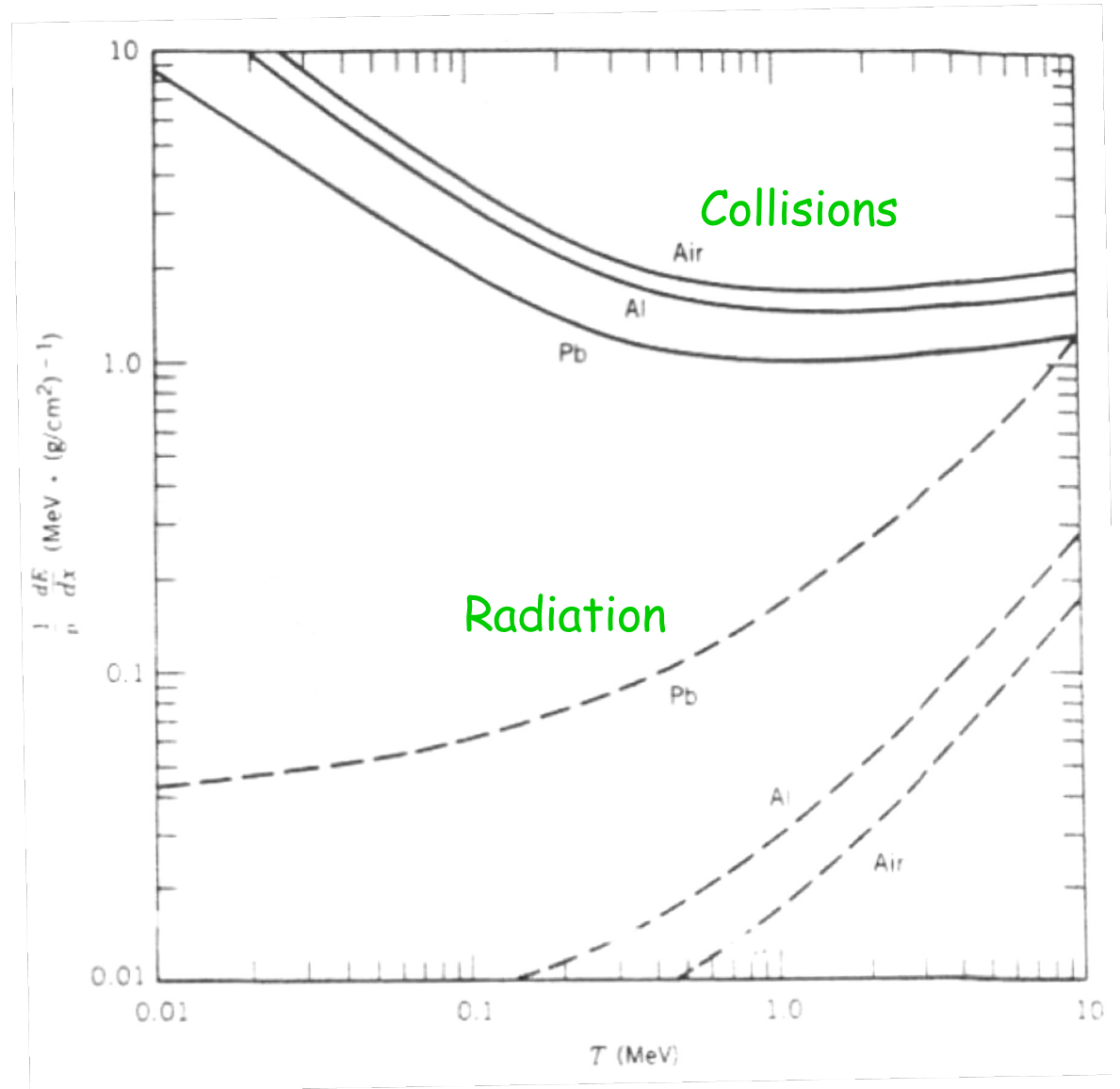
n - number of electrons per unit volume of the material, as before.

The relative importance of the two terms

$$\frac{(dE/dx)_r}{(dE/dx)_c} \approx \frac{T + m_e c^2}{m_e c^2} \frac{Z}{1600}$$

The ratio between the two contributions shows that the radiative contribution becomes important in two cases:

- for **high values of the kinetic energy** of the particle.
- for **elements with a high atomic number**.



Once again, the calculation using the formula is very complicated, and we use empirical data

Let us calculate the range of a 1 MeV beta particle in air. From the graph we read off ,

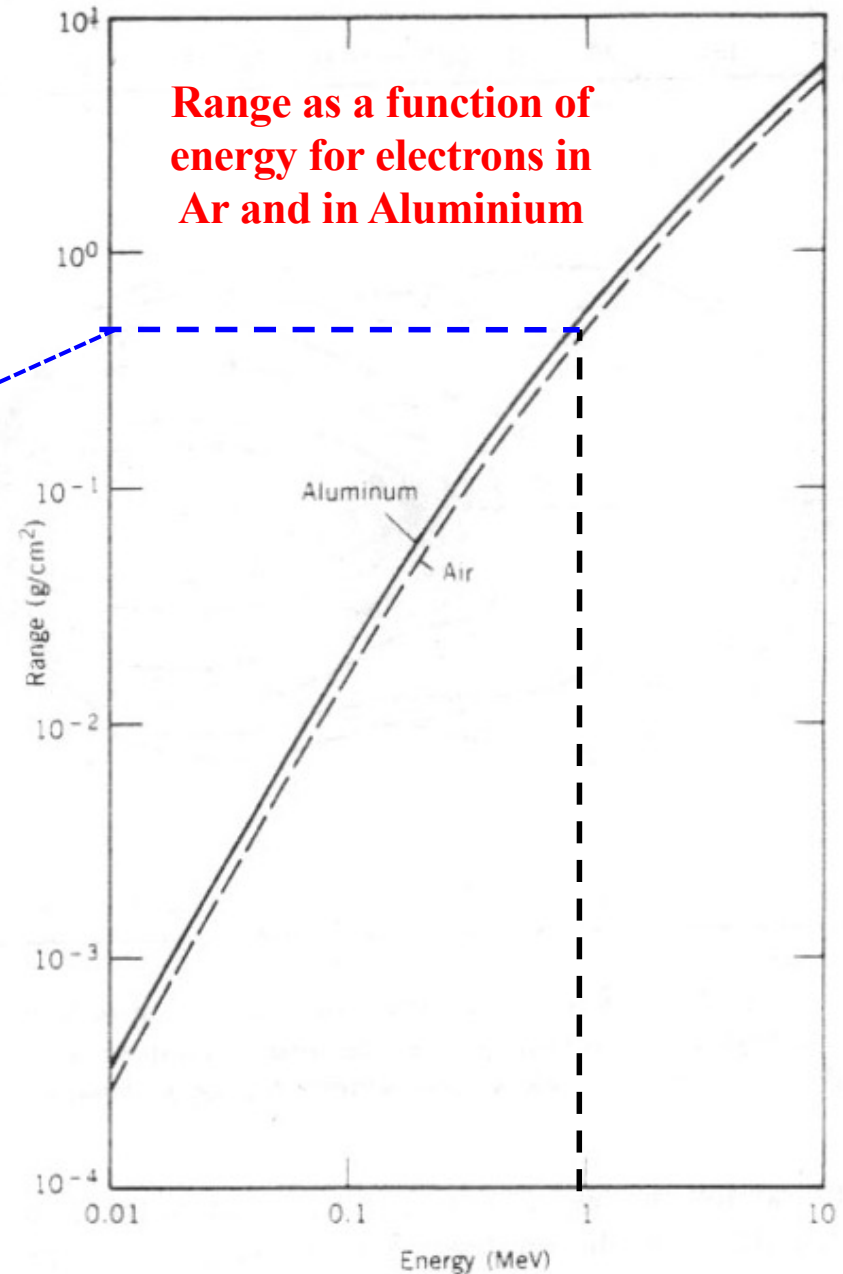
$$(\rho R)_{\text{ar}} \approx 0.6 \times 10^3 \text{ mg/cm}^2$$

And so the range is

$$R_{\text{ar}} \approx 600/1.2 = 500 \text{ cm!}$$

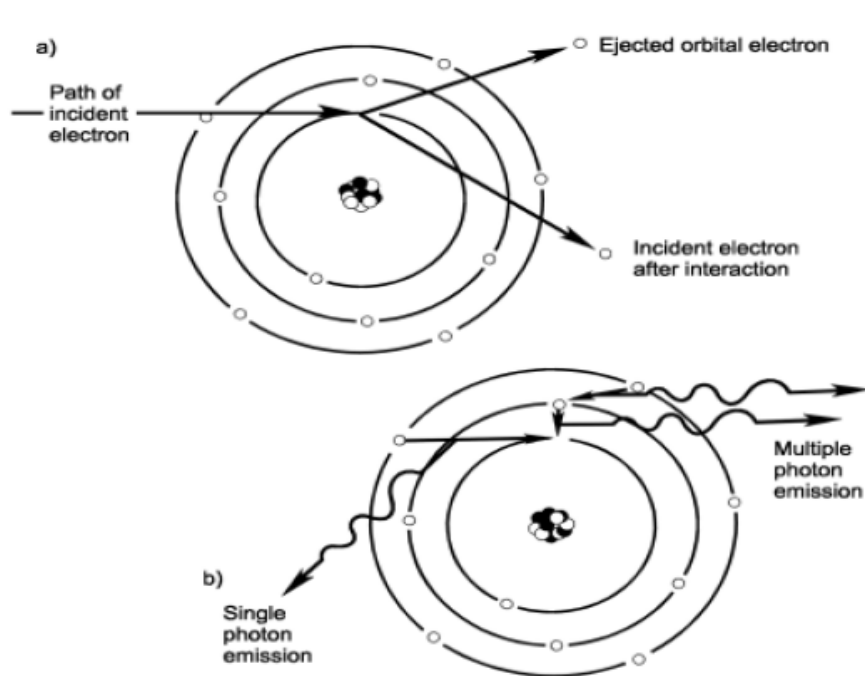
In aluminium, would the range be greater or smaller?

$$\rho_{\text{aluminium}} = 2.70 \text{ g/cm}^3 \Rightarrow R_{\text{Al}} \approx 0.22 \text{ cm}$$

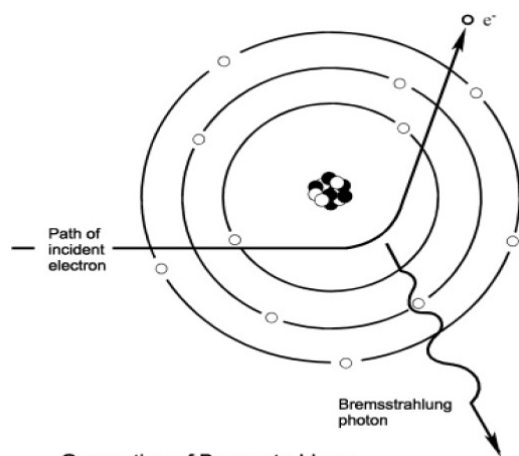


INTERACTION OF BETA PARTICLES WITH MATTER (SUMMARY)

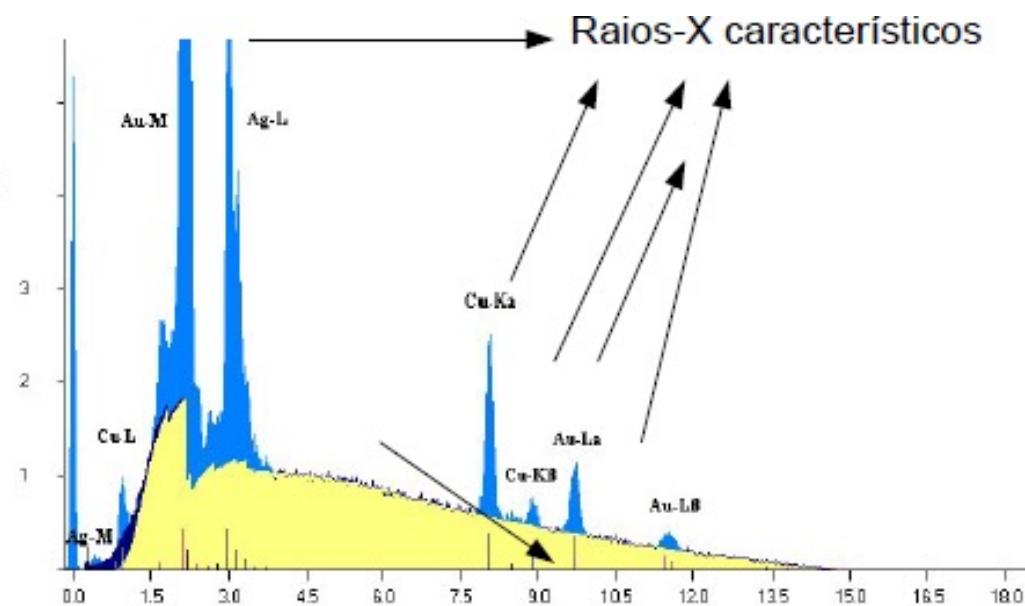
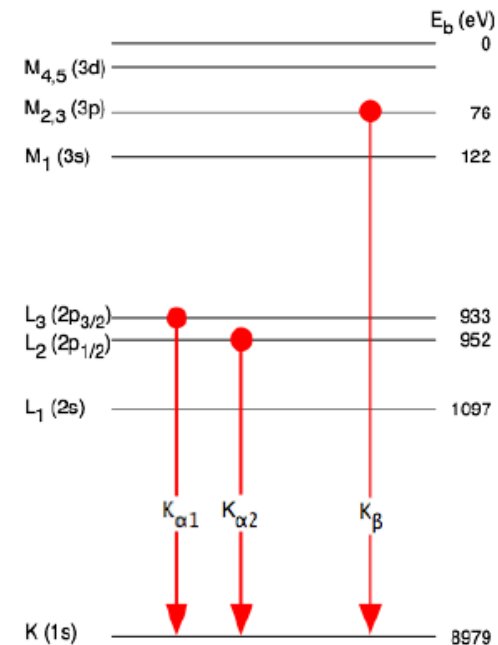
- Beta particles (electrons and positrons) have interactions of the type *interaction of light, charged particles with matter*.
- They interact fundamentally in two ways: through **COLLISIONS** with atomic electrons, or through the emission of **RADIATION** as a result of being accelerated, either by repulsion from electrons or attraction from nuclei.
- In these collisions with atomic electrons, electrons may be ejected from the atoms, with the consequent emission of X-rays when electrons from higher shells fill the vacancies left behind – **CHARACTERISTIC X-RAYS**.
- The *braking radiation* or *bremsstrahlung* is emitted because, as is known from electromagnetism, accelerated electric charges emit electromagnetic radiation **(this is the basic principle behind X-ray production in X-ray tubes!)**
- Characteristic X-rays have a discrete spectrum, while *bremsstrahlung* has a continuous spectrum. The two frequently appear superimposed.
- Positrons have another possibility: **they can annihilate with electrons present in the medium, producing two photons**. Since this collision tends to occur once the positrons have already come to rest, each photon has an energy approximately equal to the mass of an electron/positron: 511 keV.



Generation of characteristic radiation
a) Ejection of orbital electrons
b) Emission of characteristic photons



Generation of Bremsstrahlung



Espectro contínuo de *bremsstrahlung*

3. Gamma radiation

When a beam of photons passes through a material, it can undergo three types of interaction:

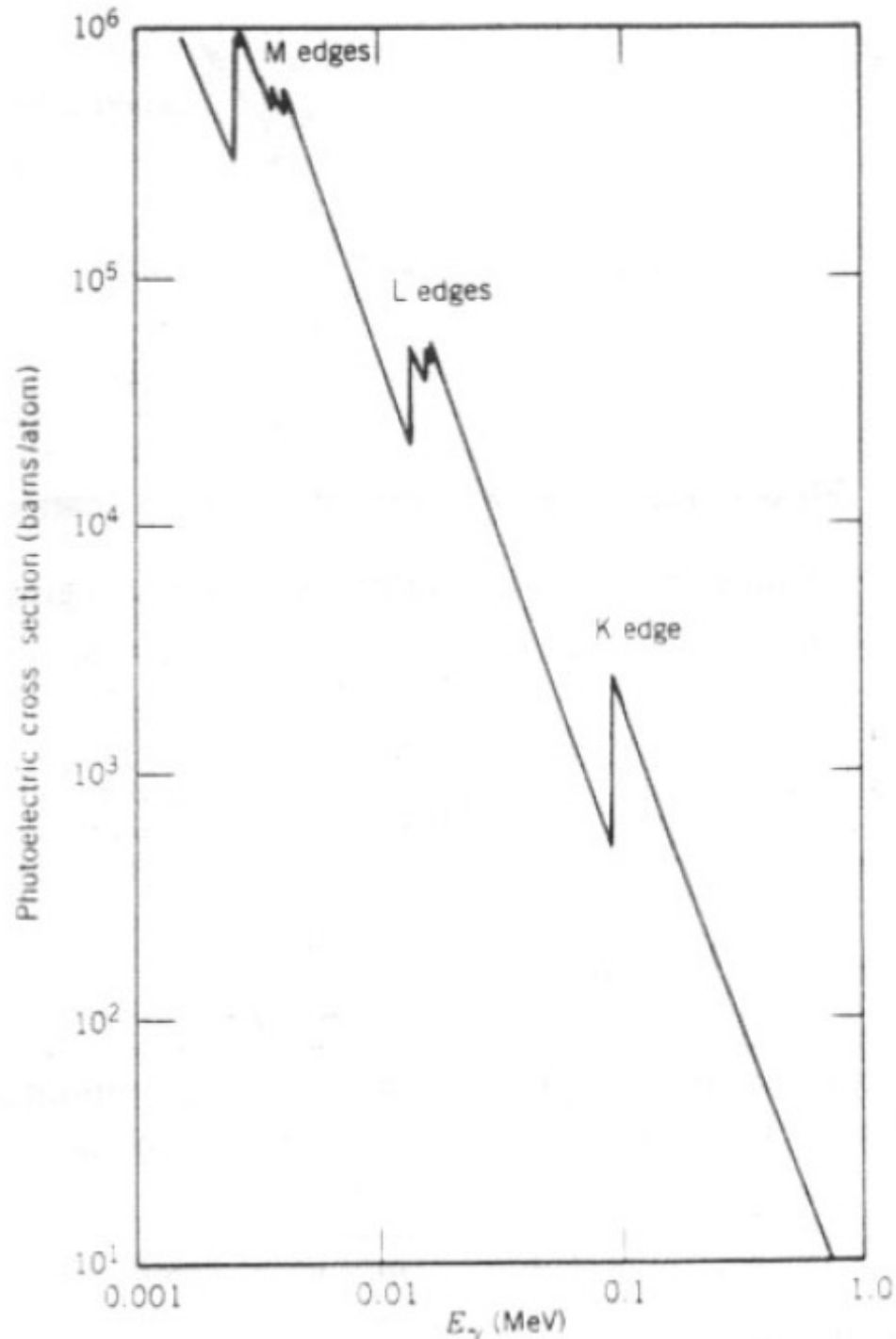
a) **Photoelectric effect** - the photon collides with an electron, knocking it out of the atom, which absorbs the energy

$$T_e = E_\gamma - B_e$$

where T_e is the kinetic energy of the electron, E_γ is the photon energy and B_e is the binding energy of the electron.

The probability for photoelectric absorption is very difficult to calculate. It is particularly important for low-energy photons (of the order of 100 keV). It increases rapidly with atomic number (approximately as Z^4) and decreases rapidly as the photon energy increases (approximately as $(1/E_\gamma)^3$).

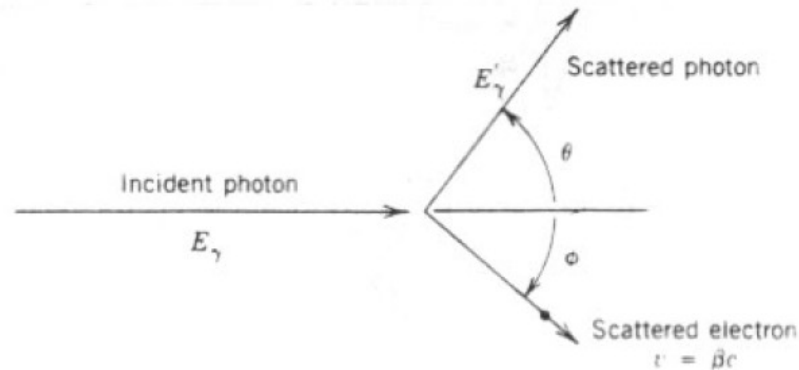
When moving from one electron shell to another, there can be a very large variation due to the change in binding energy.



Photoelectric cross-section in **picobarns**. The peaks correspond to the electron shells. The K shell, for example, has an ionisation energy of 88 keV. As we move towards more inner shells, it becomes progressively harder to remove an electron, since it is more tightly bound.

The figure shows the collision cross-section for the case of lead. To convert the cross-section (discussed further on) into a linear absorption coefficient (also discussed further on), one multiplies by 0.0333, and the coefficient comes out in cm^{-1} .

b) **Compton effect** – scattering by “free” electrons



The geometry of the Compton effect.

The absorption of photons per scattering electron (i.e. the number of photons that disappears from the initial beam per electron) is given by

$$\sigma_c = \frac{\pi r_0^2}{\alpha} \left\{ \left[1 - \frac{2(\alpha + 1)}{\alpha^2} \right] \ln(2\alpha + 1) + \frac{1}{2} + \frac{4}{\alpha} - \frac{1}{2(2\alpha + 1)^2} \right\}$$

Where α is the photon energy in units of the electron rest energy, i.e. $\alpha = E_\gamma/m_e c^2$, r_0 is a parameter with dimensions of length (unfortunately called the classical electron radius) given by

$$r_0 = \frac{e^2}{4\pi\epsilon_0 m c^2} \approx 2.818 \text{ fm}$$

c) **Pair production** – the last process by which the photon can disappear is the one in which an electron-positron pair is created.

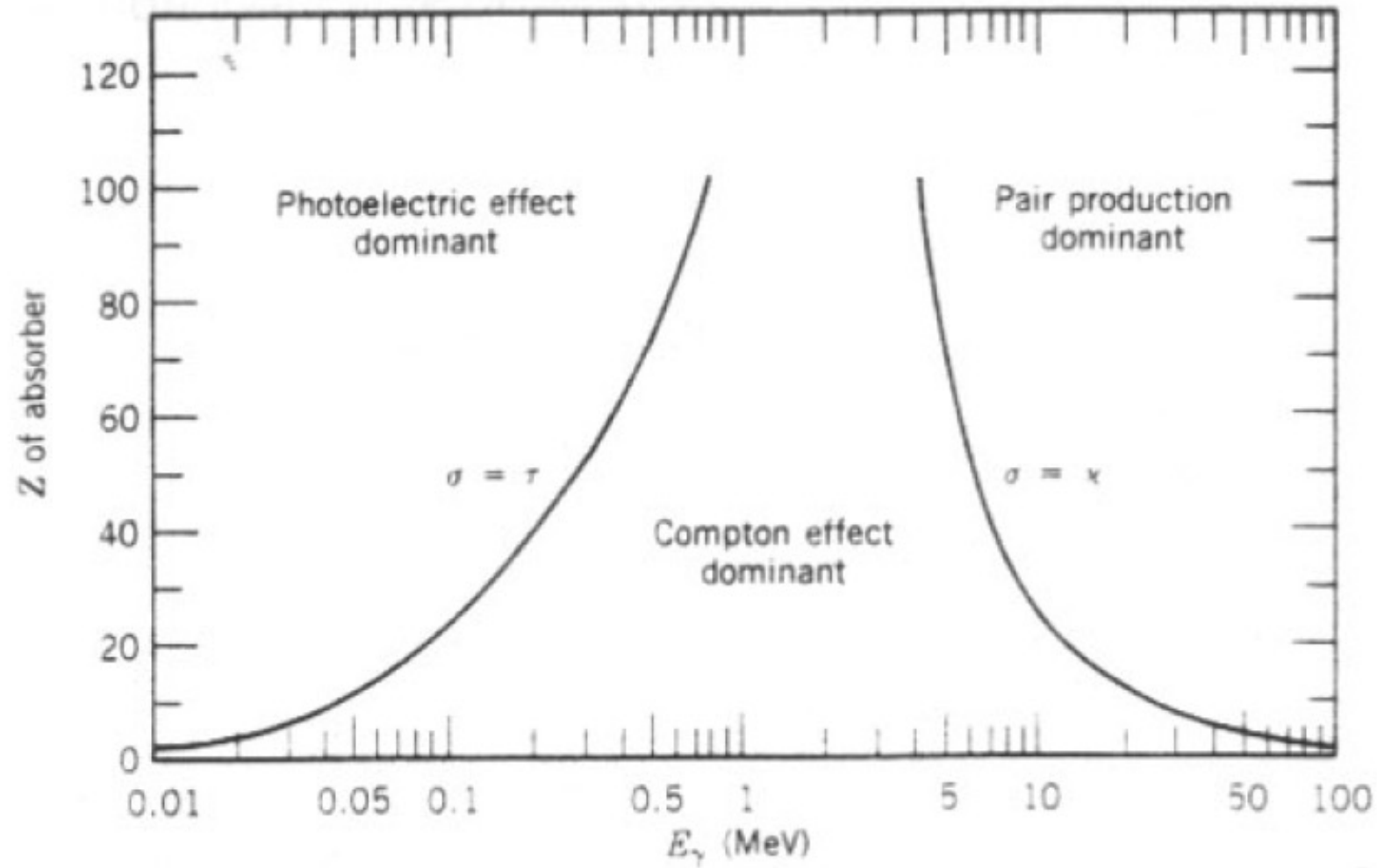
The energy balance of the process is

$$E_{\gamma} = T_{+} + m_e c^2 + T_{-} + m_e c^2$$

where T_{+} and T_{-} are the kinetic energies of the positron and the electron, respectively.

As in the case of the photoelectric effect, this process requires the presence of an atom for linear momentum to be conserved. There is an obvious energy threshold that must be exceeded for pair creation to occur, which is $2m_e c^2$, i.e. approximately 1.022 MeV. This process becomes the dominant one for energies above 5 MeV.

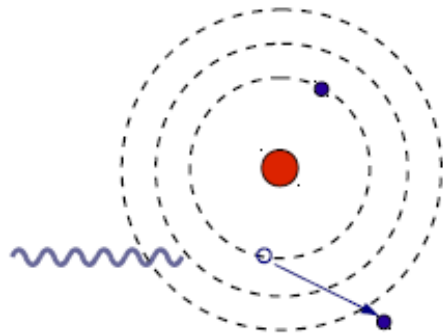
The three gamma interactions and the regions where they dominate, as a function of two variables: the energy of the incident photon and the atomic number of the target.



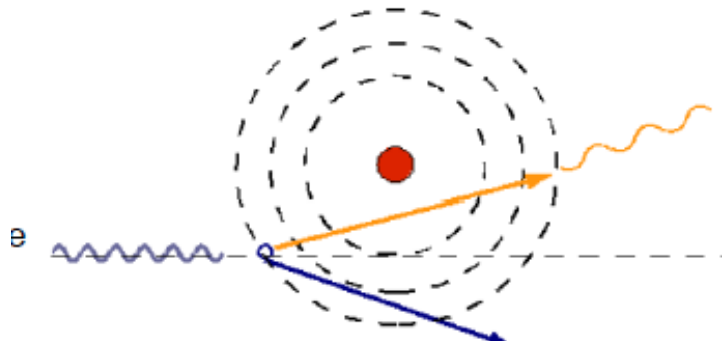
INTERACTION OF PHOTONS WITH MATTER (SUMMARY)

- Photons (X-ray or gamma) can interact/lose energy in three fundamental ways (for the range of ionising-radiation energies generally considered).
- The **PHOTOELECTRIC EFFECT** is relevant for photons of relatively low energy (*below about 100 keV*). It is more likely to occur the higher the atomic number of the element (the probability of the photoelectric effect grows with $\sim Z^4$). It corresponds to the ABSORPTION of a photon by an atom, with the subsequent ejection of an electron (ionisation), which then goes on to deposit energy in the material.
- The **COMPTON EFFECT** is relevant for intermediate energies (*between ~ 0.1 MeV and ~ 1 MeV*). It is essentially independent of the atomic number Z . It corresponds to a “collision” with a free electron, with the initial photon emerging with altered direction and energy. The electron also emerges with a different energy, and both particles go on to continue depositing energy.
- The **PAIR PRODUCTION EFFECT**, which only begins to occur at energies above two electron masses (1.022 MeV). It varies with Z^2 and corresponds to the photon giving rise to an electron-positron pair, which then go on to deposit energy.

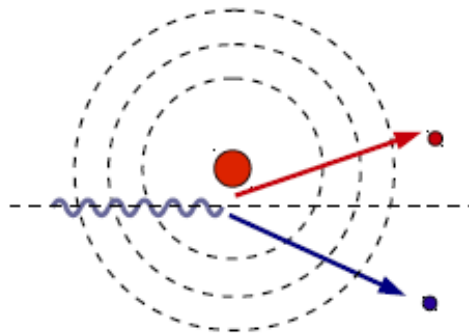
Efeito fotoelétrico



Dispersão de Compton

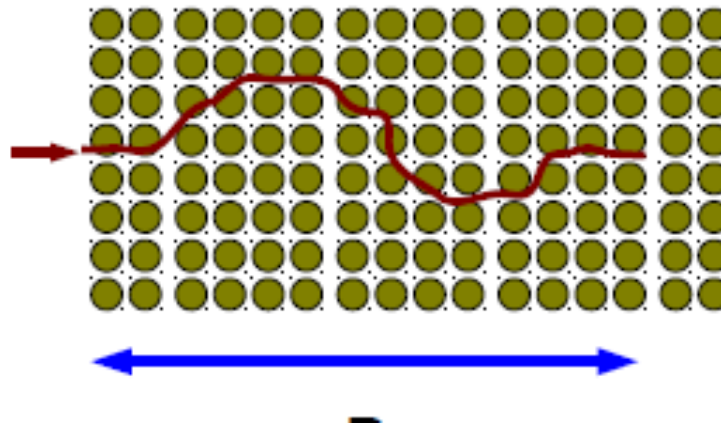


Produção de pares e^+e^-



- Since the probability of each of these effects varies with atomic number, they can have very different importance for different chemical elements.
- An initial photon of moderate energy **CAN GIVE RISE TO THOUSANDS OF SECONDARY PARTICLES, WHICH WILL DISTRIBUTE THEIR ENERGY OVER A FINITE VOLUME, RATHER THAN JUST ALONG A LINE.**
- The “range” of the radiation is a definition that is **different for photons**: the initial photon quickly disappears. The “range” corresponds to the distance **AVERAGE** at which energy deposition occurs (“average” because we are talking about a stochastic phenomenon – two simulations do not produce exactly the same outcome).

- **Electrons, positrons and especially photons produce SHOWERS of secondary particles, and the energy deposition spreads over a larger volume of space the greater the energy of the initial particle.**
- **The “range” is thus often taken as the maximum size (on average) of these showers, rather than as the distance actually travelled by the initial particle.**



Cherenkov radiation

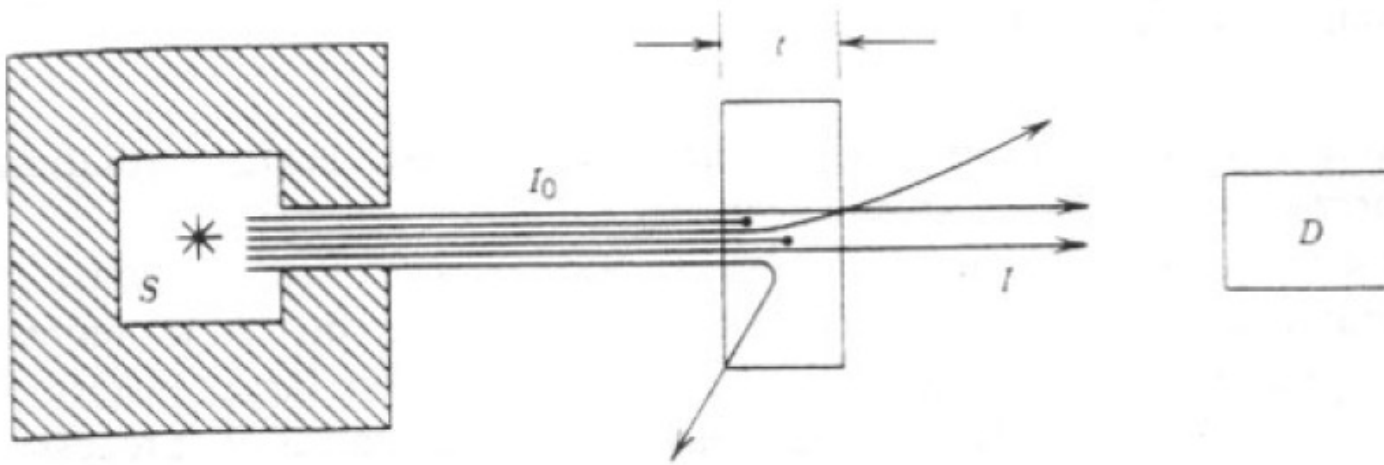


Cherenkov radiation is electromagnetic radiation that is emitted when a charged particle (such as an electron, for example) passes through a dielectric medium at a speed greater than the speed of light in that medium. The charged particles polarise the medium, and when the molecules return to the ground state they emit electromagnetic radiation.

It is the light seen in nuclear reactors.

Attenuation coefficients

Consider a beam of electromagnetic radiation incident on a target of thickness t . The radiation that passes through the target is detected by detector D.



$$\frac{dI(x)}{dx} = -\mu I(x) \quad \rightarrow$$

$$I(x) = I_0 e^{-\mu x}$$

In fact, the term used is mass attenuation coefficients for the linear attenuation coefficients divided by the density (in order to remove the effect of the number of electrons in the material).

Here μ is the total attenuation coefficient (the sum of the coefficients of the three processes).

$$\mu = \tau + \sigma + \kappa = \mu_{\text{tot}} + \mu_{\text{com}} + \mu_{\text{par}}$$

Attenuation coefficients for photons are also obtained from empirical data.

What is the attenuation of a 1 MeV beam passing through 1 cm of Al?

$$(\mu/\rho)_{\text{alu}} \approx 0.08 \text{ cm}^2/\text{g}$$

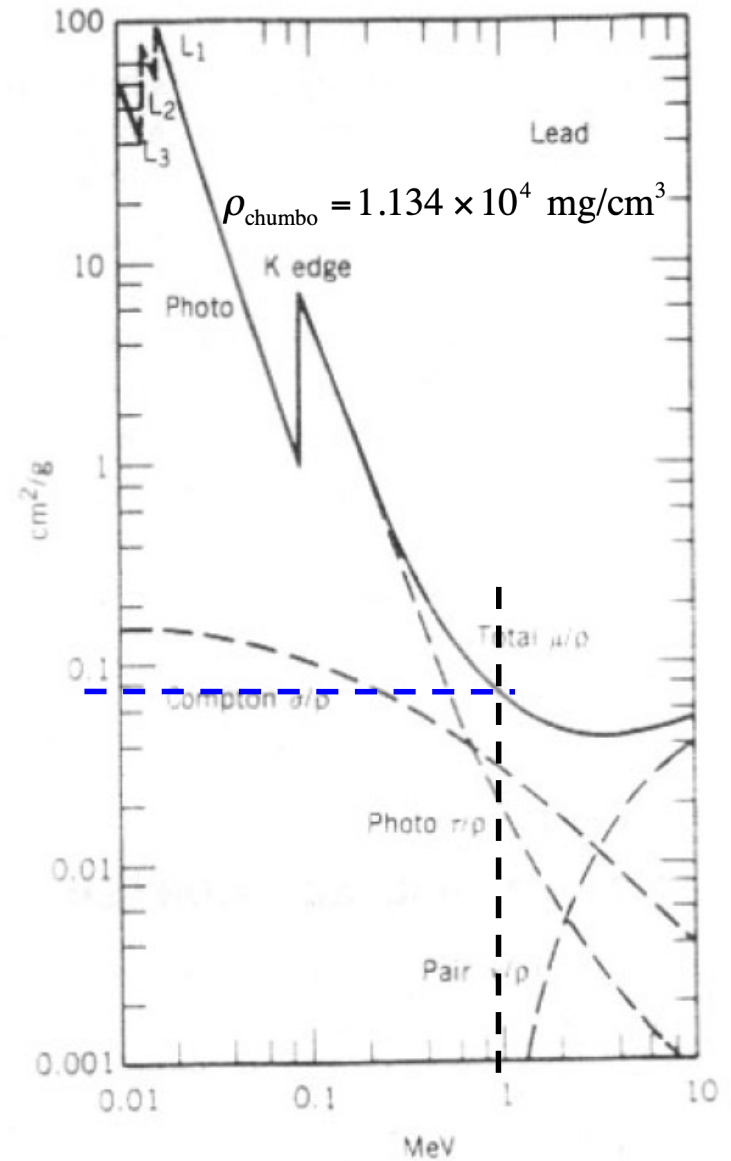
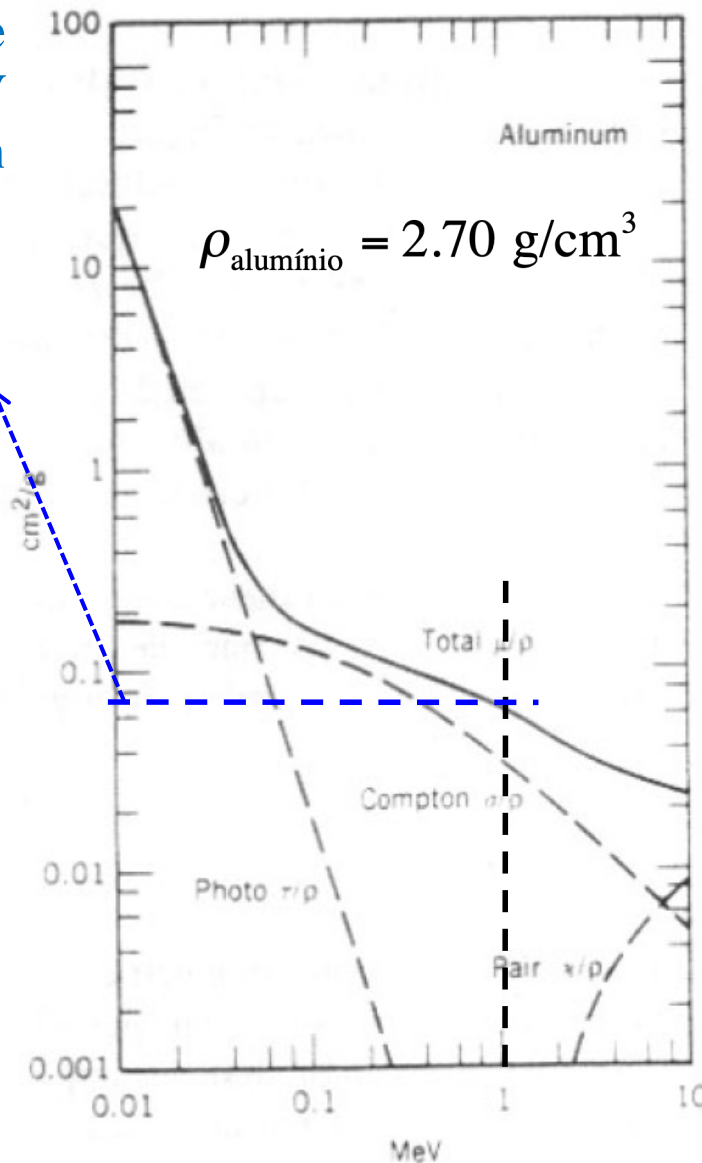


$$\mu_{\text{al}} \approx 0.216 \text{ cm}^{-1}$$



$$I(x) = I_0 e^{-0.216 \times 1} = 0.81 I_0$$

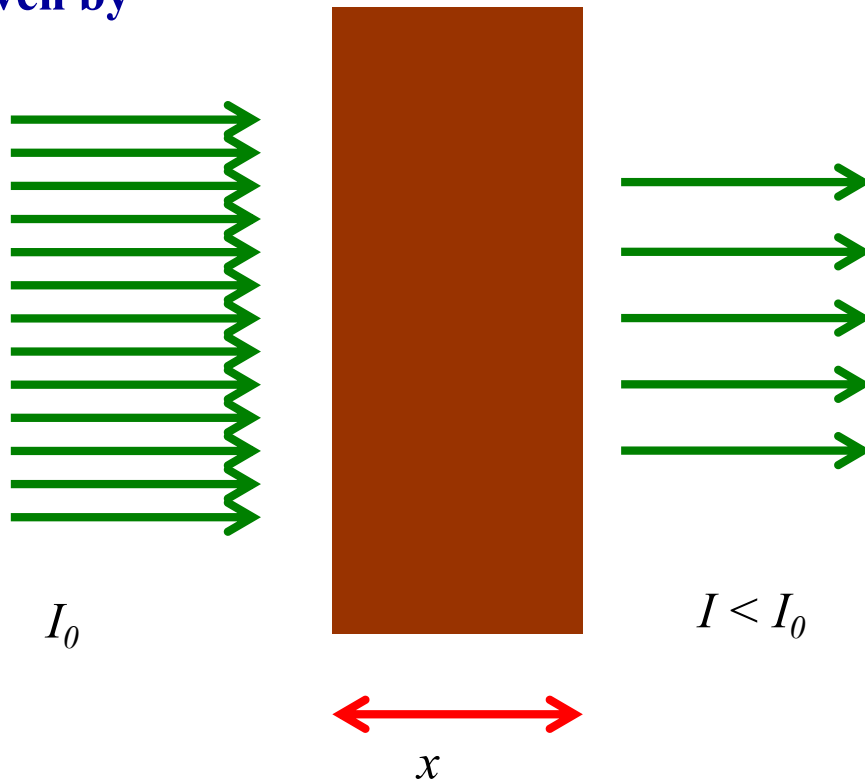
What would the result be for lead?



LAMBERT-BEER LAW

(ATTENUATION OF PHOTON BEAMS)

A beam of photons, on penetrating a given material medium, will undergo a series of interactions. Its initial intensity, I_0 , will be attenuated exponentially as it travels a distance x through the material medium. The attenuation coefficient, μ , depends on the material medium in question and on the energy of the radiation: the final intensity, I , is given by



$$I = I_0 e^{-\mu x}$$

- The attenuation coefficients μ have units of **INVERSE DISTANCE**, usually cm^{-1} .
- It is very common to find tabulated values of the *mass attenuation coefficients* (*mass attenuation coefficients*), which correspond to the value of μ divided by the mass density of the material, μ/ρ (units: cm^2/g).
- The coefficients μ have contributions from each of the three effects experienced by photons,

$$\mu = \mu_{\text{Photoelectric}} + \mu_{\text{Compton}} + \mu_{\text{Pairs}}$$

- Higher values of μ correspond to greater attenuation – more photons are absorbed by the material, and more energy is deposited.
- Radiological protection – materials with the highest possible μ are chosen! Since μ increases with Z , denser and heavier elements (typically metals) are chosen.

ATTENUATION COEFFICIENTS FOR COMPOUND SUBSTANCES

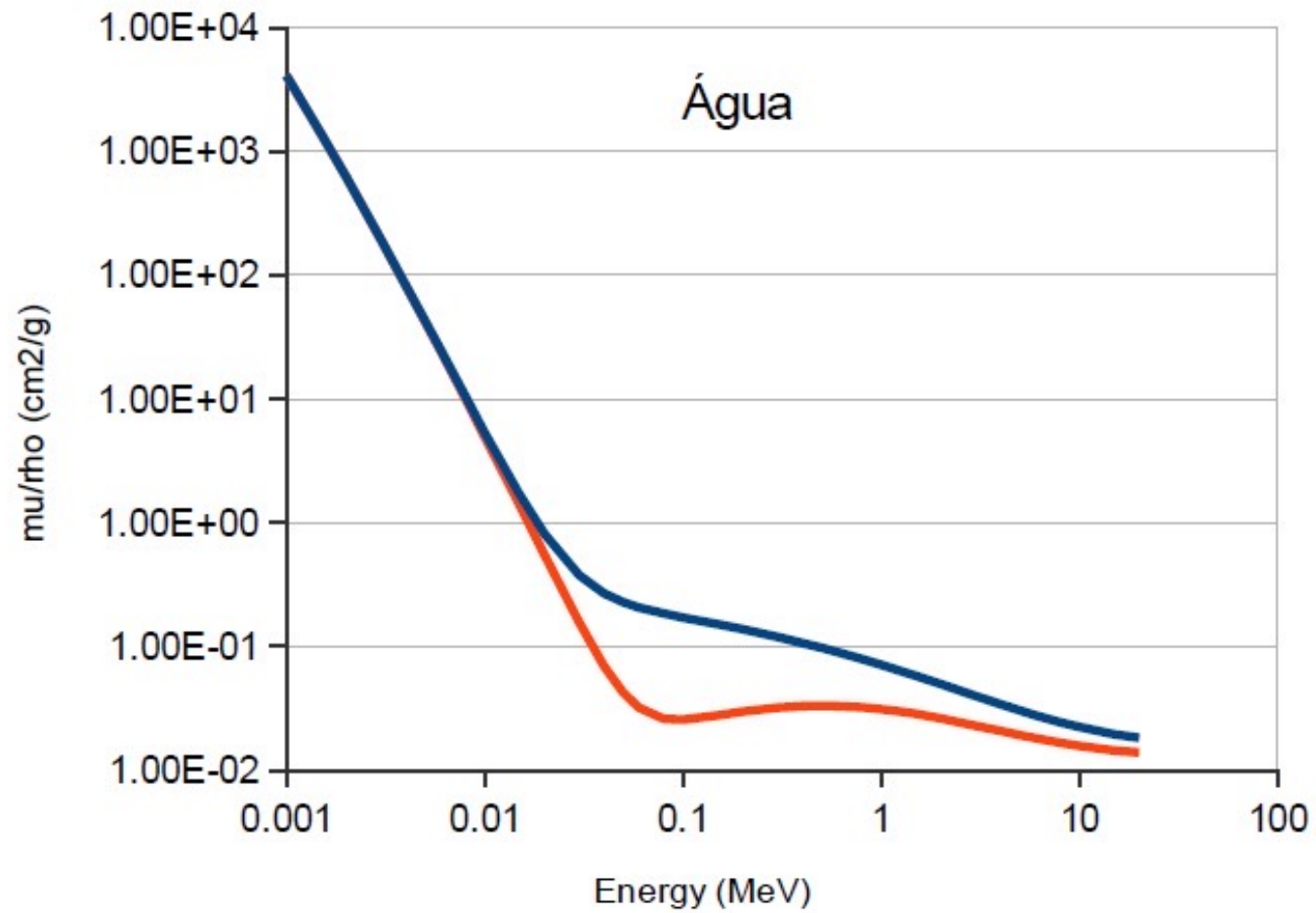
Photons interact with each atom present in material media. The mass attenuation coefficient of a compound medium is thus a combination of the mass attenuation coefficients of each chemical element present, weighted by the *mass fractions* of each element:

$$\left(\frac{\mu}{\rho}\right)_{\text{mistura}} = w_1 \left(\frac{\mu}{\rho}\right)_1 + w_2 \left(\frac{\mu}{\rho}\right)_2 + \dots = \sum_{i=1}^N w_i \left(\frac{\mu}{\rho}\right)_i$$

Note that the attenuation coefficient of each element appears divided by its respective mass density. The *mass fractions* are given by

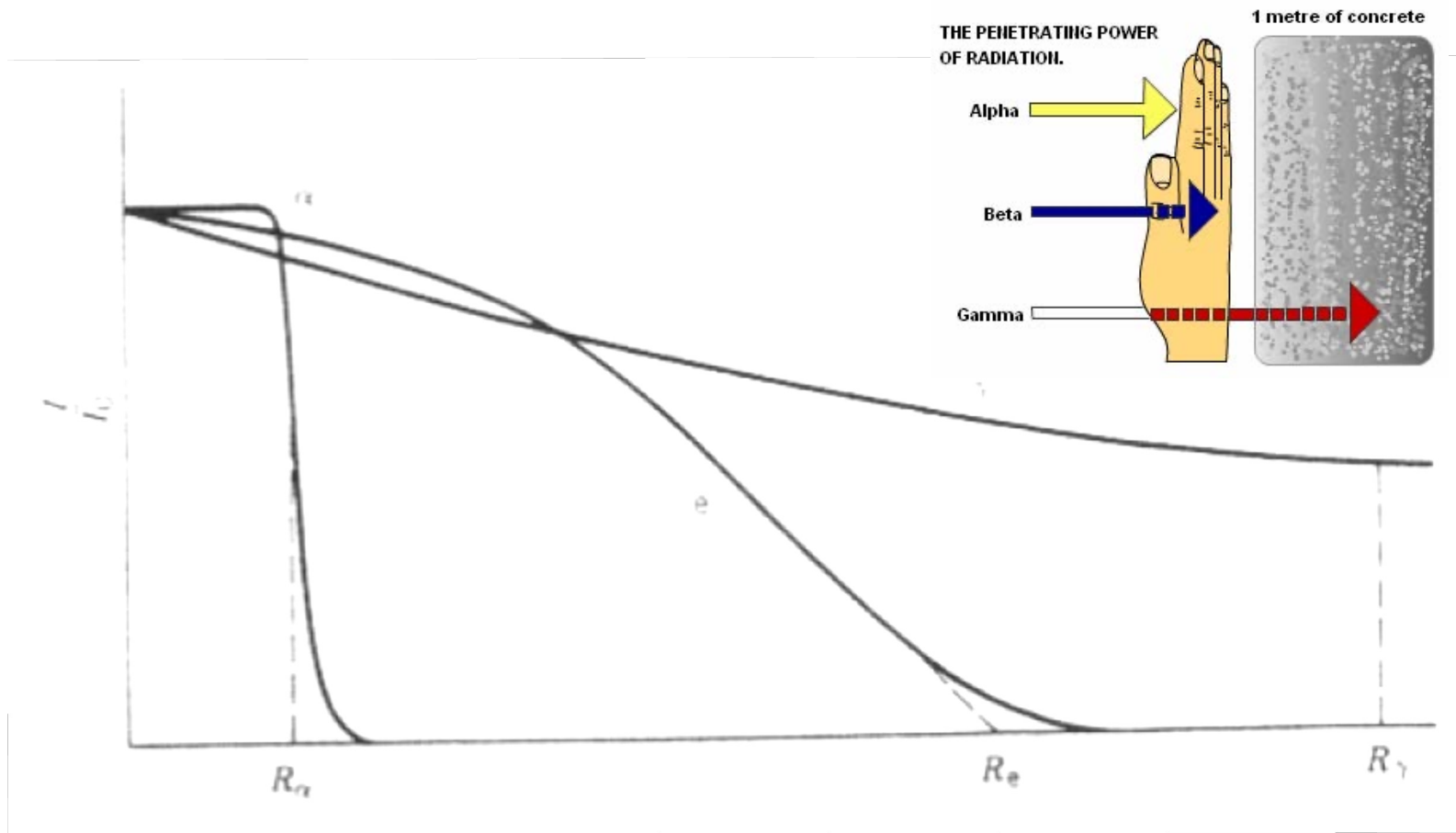
$$w_i = \frac{\text{massa do elemento } i}{\text{massa total da mistura}}$$

There are tabulated values for mixtures, which are calculated in this way.



Mass attenuation coefficient for water, over a range of energies

I/I_0 for alpha, beta and gamma radiation. The ranges are defined identically for alpha and gamma radiation – the value of R is defined as the point at which the beam intensity is reduced to half



The horizontal scale is not linear. The range for gammas can be 10,000 times greater than the range for alphas.

For electrons it is customary to define the range as the extrapolation of the linear part of the curve, as shown in the figure.